

Cálculo Integral em \mathbb{R}^n .

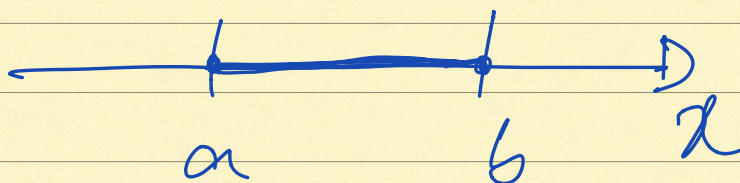
Áreas em \mathbb{R}^2

CDI-I: Teorema Fundamental do Cálculo.

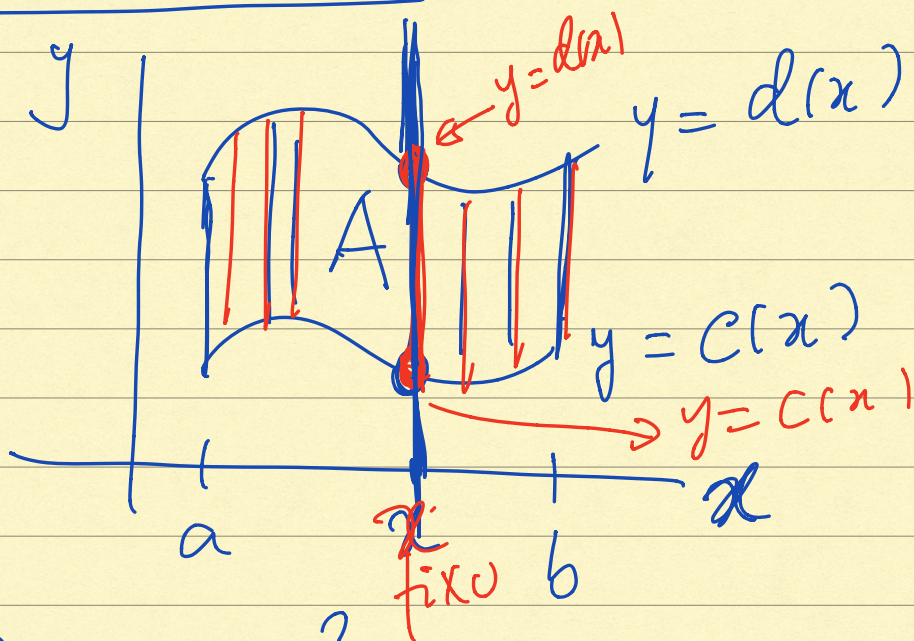
$$\int_a^b f'(t) dt = f(b) - f(a)$$

Se $f'(t) = 1$ $f'(x) = 1$

$$\int_a^b 1 dx = b - a$$



Área em \mathbb{R}^2 ; $c, d: [a, b] \rightarrow \mathbb{R}$



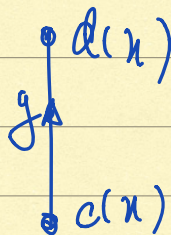
$$A = \left\{ (x, y) \in \mathbb{R}^2 : a \leq x \leq b; \underbrace{c(x) \leq y \leq d(x)} \right\}$$

$$\text{área de } A = \int_a^b \underbrace{(d(x) - c(x))}_{\text{comprimento do segmento de reta}}$$

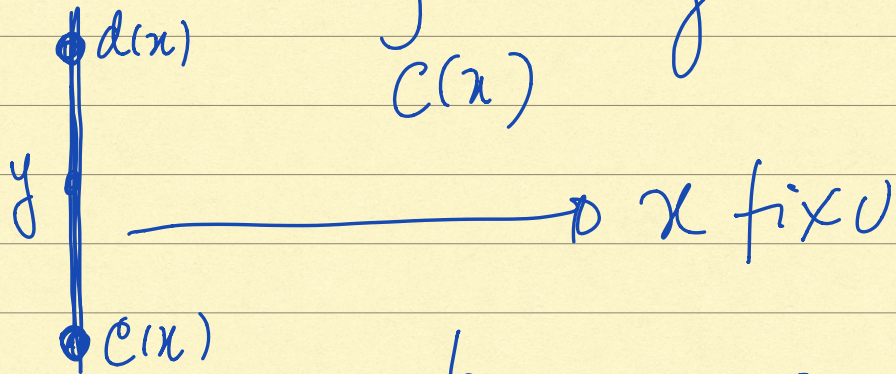
comprimento
do segmento de
reta

área \equiv integral de comprimentos

$$d(x) - c(x)$$



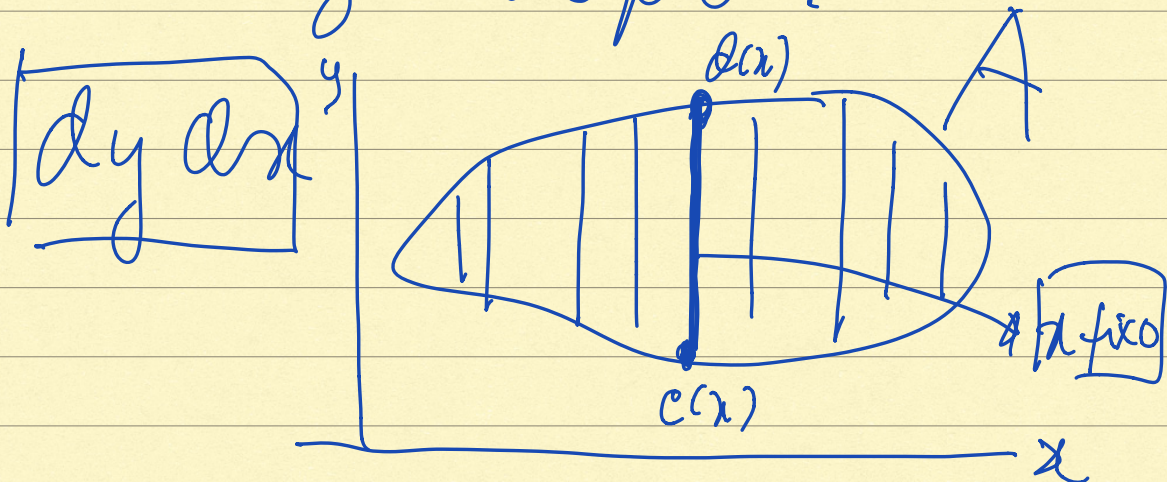
$$d(x) - c(x) = \int_{c(x)}^{d(x)} 1 dy$$

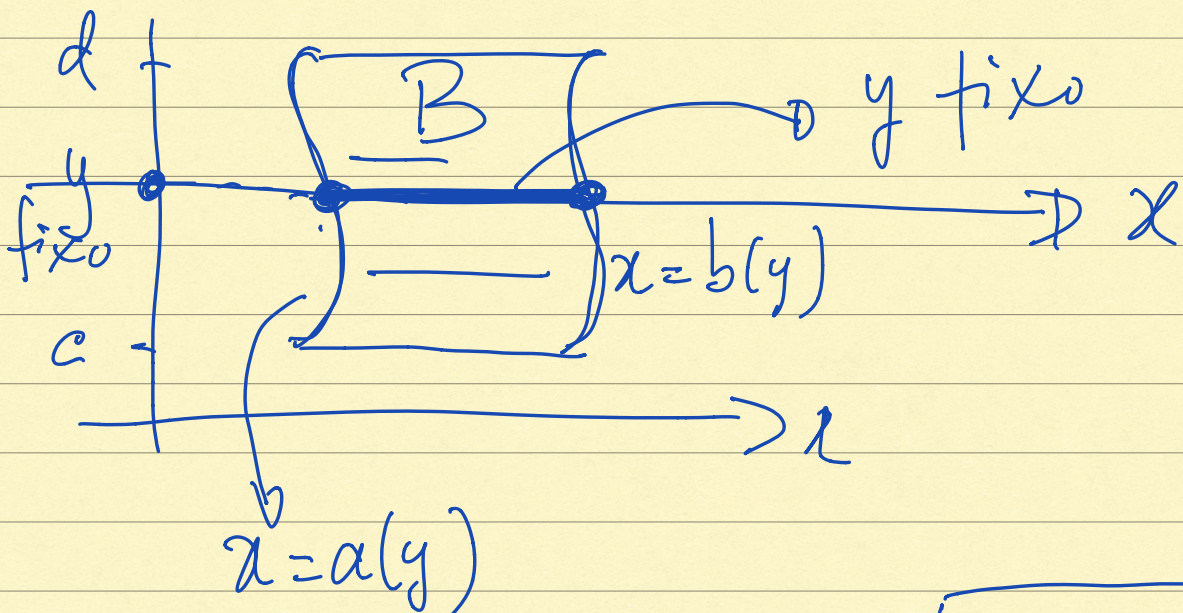


$$\text{area de } A: \int_a^b (d(x) - c(x)) dx =$$

$$= \int_a^b \left(\int_{c(x)}^{d(x)} 1 dy \right) dx \quad \left[\begin{array}{l} x \text{ fixu} \\ \text{entre} \\ a \text{ e } b \end{array} \right]$$

integral duplo!



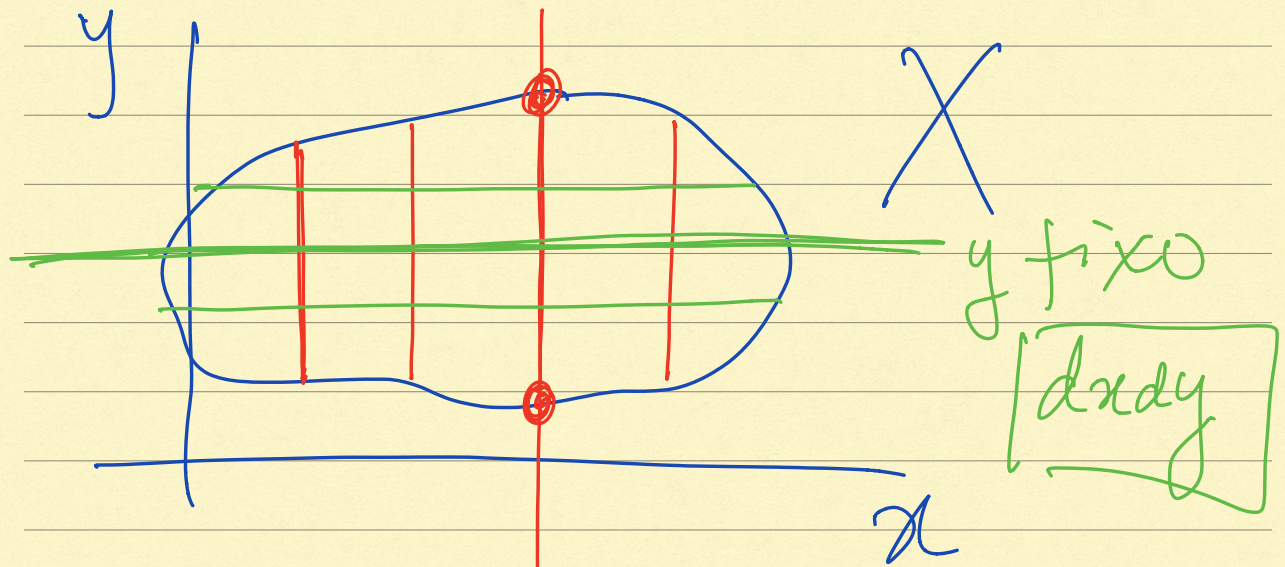


$$B = \left\{ (x, y) \in \mathbb{R}^2 : c \leq y \leq d; \overbrace{a(y) \leq x \leq b(y)} \right\}$$

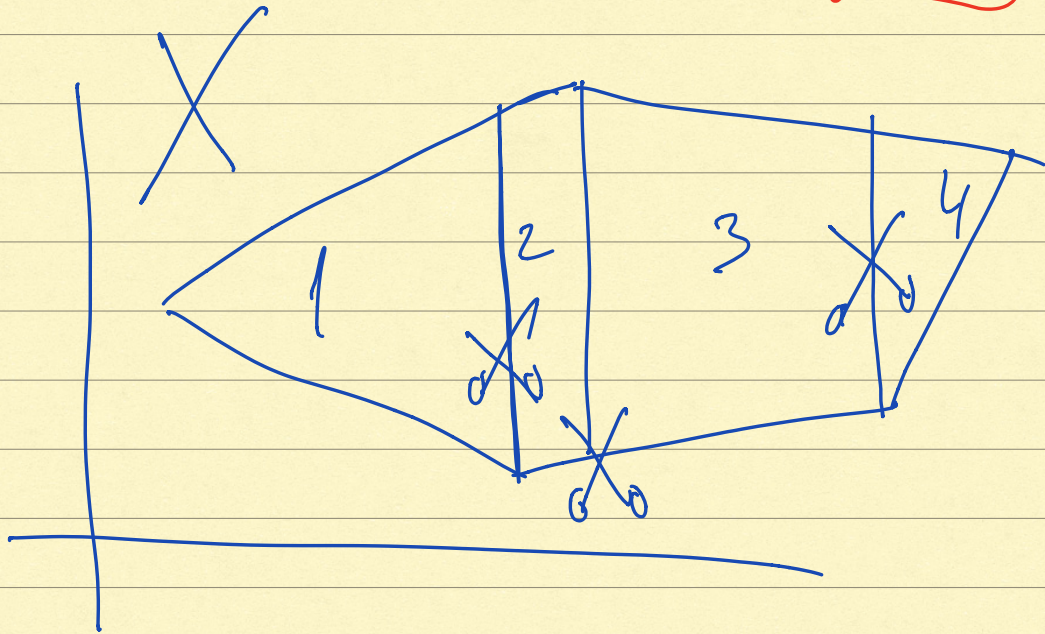
$$\text{área de } B: \int_c^d (b(y) - a(y)) dy$$

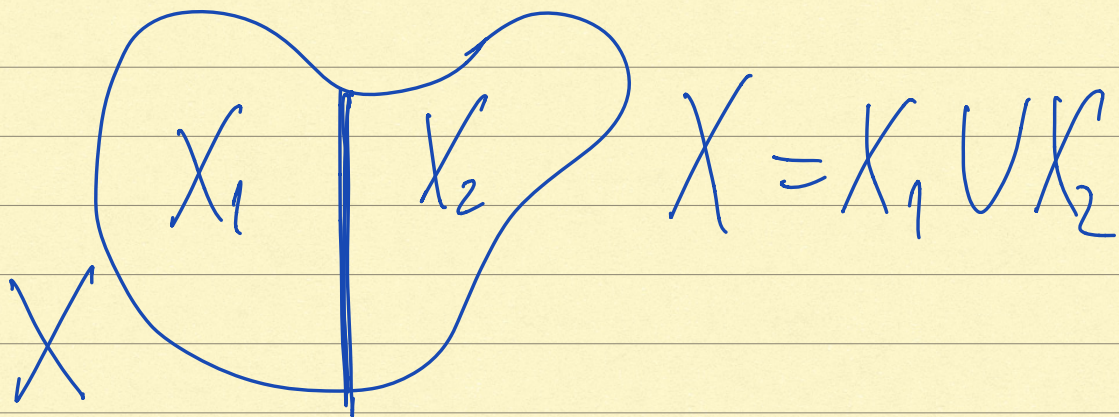
$$= \int_c^d \left(\int_{a(y)}^{b(y)} 1 dx \right) dy$$

$$\boxed{dx dy} \quad y \text{ fixo} \rightarrow x(y)$$

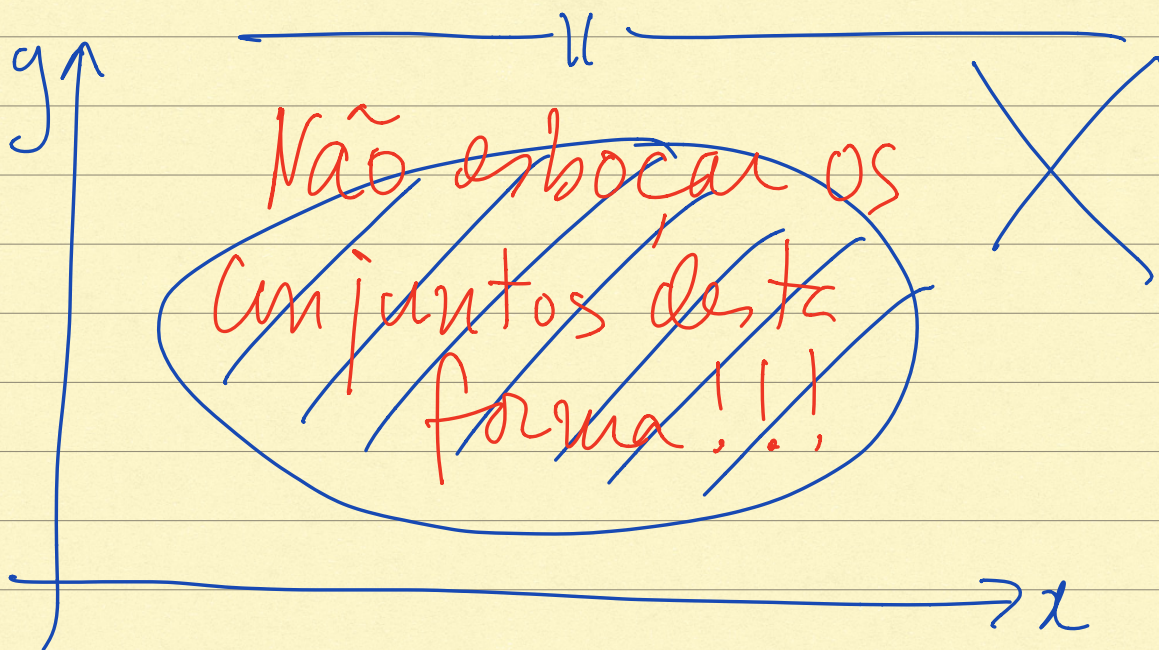


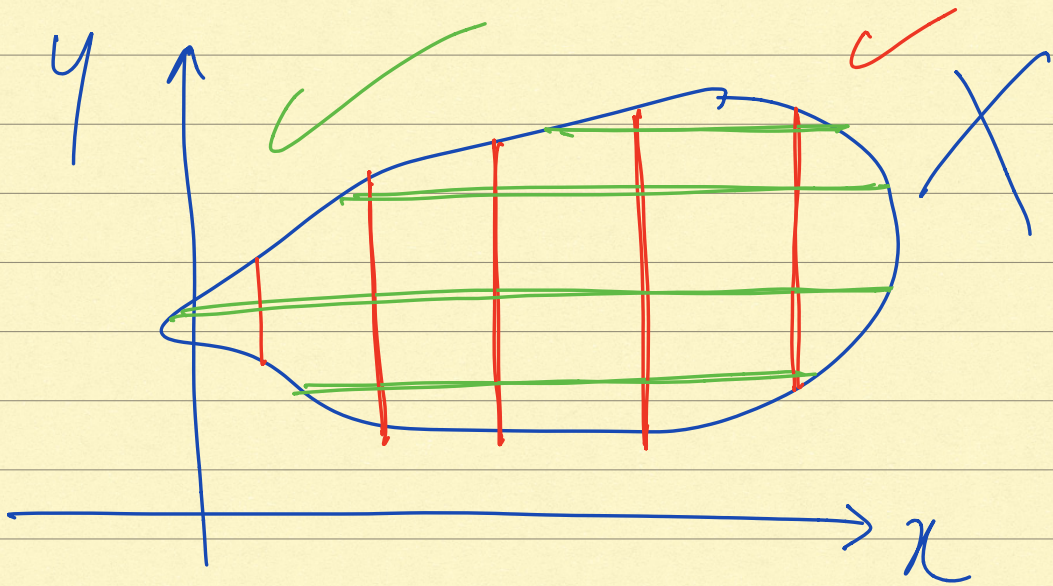
x fixo
 $dy dx$



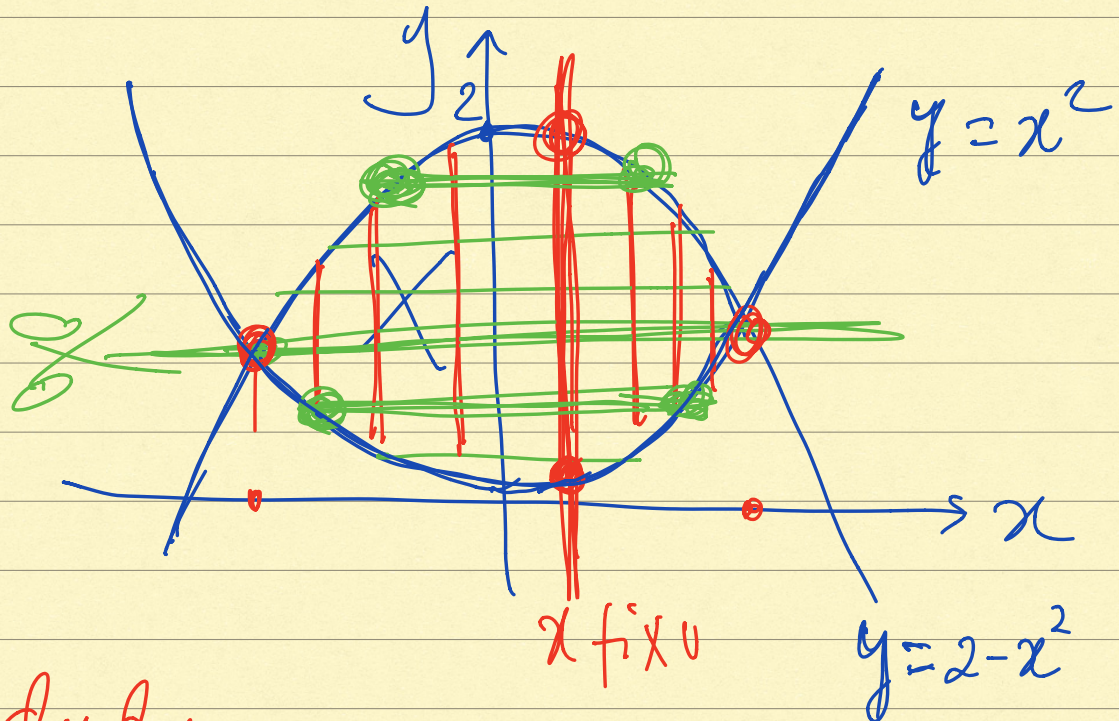


área de $X \equiv$ área de X_1
 + área de X_2 .





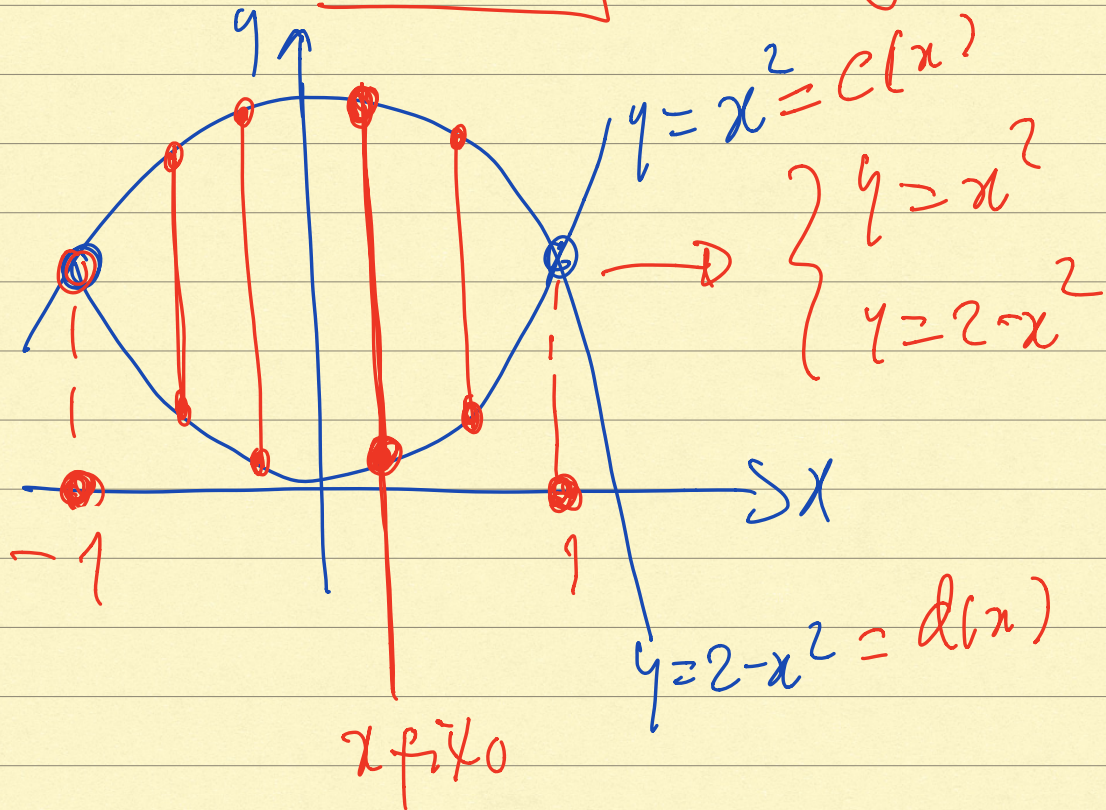
Exemplo: $X = \{ (x, y) \in \mathbb{R}^2 : x^2 < y < 2-x^2 \}$



$dy dx$
1 parte!

$dx dy$
2 partes

dydx: x fixo $\rightarrow y(x)$



$$x^2 = 2 - x^2 \Leftrightarrow 2x^2 = 2$$

$$\Leftrightarrow x^2 = 1$$

$$\Leftrightarrow x = -1 \vee x = 1$$

x fixo $[-1, 1]$.

$$\text{Área de } X : \int_{-1}^1 \left(\int_{x^2}^{2-x^2} 1 \, dy \right) dx$$

$$= \int_{-1}^1 (2 - x^2 - x^2) dx$$

$$= \int_{-1}^1 (2 - 2x^2) dx =$$

$$= 2 \int_0^1 (2 - 2x^2) dx$$

$$= 2 \left(2 - \frac{2}{3} \right) = \frac{8}{3}$$

$dxdy$ y fixa $\rightarrow x(y)$